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Reg. No. : .....

Name : .....

Fifth Semester B.Sc. Degree Examination, December 2017
First Degree Programme under CBCSS
MATHEMATICS
Core Course
MM 1541 : Real Analysis – I
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – 1

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. State Bernoulli's inequality.
2. If $S = \{1 - (-1)^n : n \in N\}$, find inf S.
3. Give an example of a set in R such that its supremum exists but has no lower bounds.
4. State Squeeze theorem for sequence of real numbers.
5. Give an example of an unbounded sequence that has a convergent subsequence.
6. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ is convergent.
7. Find all cluster points of the set $\left\{\frac{1}{n} : n \in N\right\}$.
8. Show that $\lim_{x\to 0} x^{3/2} = 0$, for $x > 0$.
9. State Bolzano-Weierstrass theorem.
10. State Cauchy criterion for convergence of a series.

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SECTION – 2

Answer **any 8** questions from this section. **Each** question carries **2** marks.

11. If $ab > 0$ then show that either (i) $a > 0$ and $b > 0$ or (ii) $a < 0$ and $b < 0$.

12. Determine the set $A = \{x \in R : x^2 + x > 2\}$.

13. State completeness property of R and justify with an example.

14. Define nested intervals and give an example for the same.

15. Show that $\lim \frac{1}{n} = 0$.

16. Show that a convergent sequence is bounded.

17. Establish the convergence of the sequence (x_n), where $x_n = \frac{n}{2^n}$.

18. Show that a Cauchy sequence of real numbers is bounded.

19. Show that the series $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ is convergent.

20. Find the limit of the sequence $\left(1+\frac{1}{2n}\right)^n$, $n \in N$.

21. Using $\varepsilon - \delta$ definition of limit show that $\lim_{x \to c} x = c$.

22. Show that $\lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1$.

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SECTION – 3

Answer **any 6** questions from this section. **Each** question carries **4** marks.

23. Let $f(x) = \frac{2x^2 + 3x + 1}{2x - 1}$, where $2 \le x \le 3$. Determine a constant M such that $|f(x)| \le M$ for all x satisfying $2 \le x \le 3$.

24. State and prove Archimedean property.

25. Let (x_n) be a sequence of positive real numbers such that $\lim \frac{x_{n+1}}{x_n} = L$. If $L < 1$, then show that (x_n) converges and $\lim x_n = 0$.

26. State and prove Monotone subsequence theorem.

27. Show that the geometric series $\sum_{n=1}^{\infty} r^n$ converges when $|r| < 1$.

28. State and prove Limit Comparison test.

29. Show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ is divergent.

30. Show that a number $c \in R$ is a cluster point of a subset A of R if and only if there exists a sequence (a_n) in A such that $\lim (a_n) = c$ and $a_n \neq c$ for all $n \in N$.

31. Prove that $\lim_{x \to 0} \left(\frac{\cos x - 1}{x} \right) = 0$.

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SECTION – 4

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. i) State and prove Density Theorem.
    ii) State and prove Nested Intervals Property of real numbers.

33. Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers converges to x and y respectively. Then show that X + Y and X.Y and converges to x + y and xy respectively. Under suitable conditions in $Y = (y_n)$ and y further show that the quotient X/Y converges to x/y.

34. i) State and prove Cauchy's convergence criterion of a sequence.
    ii) Show that every contractive sequence is a Cauchy sequence and is convergent.

35. i) Using $\varepsilon - \delta$ definition of limit establish that $\lim_{x \to c} \frac{1}{x} = \frac{1}{c}$.

    ii) Let $f(x) = \frac{1}{e^x + 1}$ for $x \neq 0$. Show that the right-hand limit and left hand limit of f(x) exist but they are unequal.